QUANTUM PHYSICS I - Feb. 6, 2017

Write your name and student number on **all** sheets. There are four problems in this exam. You can earn 90 points in total.

PROBLEM 1: BOHR MODEL (all 5 points)

a) In the Bohr model of the 3D hydrogen atom, which quantum numbers are necessary to specify the eigenstate of a single electron? Describe the role/interpretation of each quantum number.

b) What is the degeneracy of the energy levels of these eigenstates; in other words, how many eigenstates are there with identical energies? Briefly explain your answer.

c) Explain in a few sentences how this changes when considering multiple electrons around a nucleus. What new physical effect is present and how does this change the energy levels?

d) In the case of Lithium ($Z = 3$), what are the quantum numbers of the electrons in their ground state?

e) In the case of Potassium ($Z = 19$), what are the quantum numbers of the electrons in their ground state?

PROBLEM 2: FOURIER TRANSFORM (all 5 points)

a) What is the definition of the Fourier transform of a general function $f(x)$?

b) What special property has the Fourier transform of a symmetric function $f(x) = f(-x)$?

PROBLEM 3: DELTA-FUNCTION POTENTIAL (5+5+10 points)

Consider a delta-function potential (in one dimension) of the form

$$V = -\alpha\delta(x)\,. \tag{1}$$

a) How many bound states does this potential have? Sketch the corresponding wavefunction(s), with special emphasis on smoothness/continuity properties.

b) What is the general form $\psi(x)$ of incoming scattering states for this potential? Which parameters does it have?

c) Suppose there is a scattering state coming in from the left. Derive the transmission and reflection probabilities.

PROBLEM 4: GAUSSIAN WAVEFUNCTION (all 5 points)

Consider a wavefunction on one dimension whose spatial dependence is Gaussian, i.e.

$$\psi(x) = Ae^{-ax^2}\,, \tag{2}$$

where the constant A is fixed by normalization.

a) For which potential energy V is the above wavefunction an energy eigenstate (i.e. a stationary state)?

b) What is the wavefunction depending on momentum that corresponds to the above wavefunction depending on location, i.e. what is its Fourier transform $\psi(p)$?

c) What is the probability of measuring a momentum that is positive? Briefly explain your answer.

d) Calculate the variance of the above state with respect to the x and to the p operators.

e) Does the above state satisfy or saturate the Heisenberg uncertainty principle for x and p?

f) Suppose a measurement is made of the location of the particle, resulting in some definite and positive x_0 up to a small uncertainty δ. Sketch what the probability distributions (for x and for p) look like after such a measurement.

g) After such a measurement, what can you say about the probability of subsequently measuring a momentum that is positive? Briefly explain your answer.